COMPUTATION OF AERODYNAMIC LOADS ON HELICOPTER ROTORBLADES IN FORWARD FLIGHT, USING THE METHOD OF THE ACCELERATION POTENTIAL

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1. Summary

The paper describes an investigation concerning the question whether Prandtl's classical lifting line theory and Weissinger's extended lifting line theory (3/4-chord method) are really applicable to the aerodynamic analysis of the helicopterrotor in forward flight. Although usually a lifting line model in one form or the other is taken as the basis of rotoranalysis, no existing theory is entirely satisfactory for the analysis of the unsteady, sheared flow encountered by the blades of a helicopterrotor.
In the paper the description of the flowfield is based on the acceleration potential instead of the usual velocity potential. The use of the acceleration potential allows a relatively easy derivation of lifting line theory using a "matched asymptotic expansion" technique. The systematic treatment afforded by this approach enables one to gain a clear insight into the problems associated with lifting line models.
Not only is the method proposed in this paper able to solve these problems relatively easily, but it also offers the means to cut down computing times considerably in actual numerical calculations. The evaluation of the induced velocity in points on the blade, which requires normally a two-dimensional integration over the skewed helical vortex sheets forming the rotorwake, is reduced to a one-dimensional integration using the acceleration potential.
By means of the matched asymptotic expansion technique two numerically efficient lifting line theories, fully applicable to the helicopterblade, are derived. The first one involves errors of the order A^{-2} (where A is the aspect ratio of the blades). The second one is a more elaborate higher order method, involving relative errors of the order A^{-3}. If applied to the simpler case of the unswept wing in steady flow, these methods would reduce to Prandtl's classical method and to Weissinger's 3/4-chord method respectively.
The matched asymptotic expansion analysis yields at the same time the complete pressure distribution over the blade's surface, which is a great advantage over the existing lifting line methods.

2. Introduction

Prandtl's classical model of a lifting surface represented by a lifting line may seem to be something of the past. Lifting surface methods, whose practical use has been made feasible by the advent of fast computers, have largely superseded the lifting line theories with their inherent shortcomings. There is however one area of aerodynamics where lifting line theories are still in general use: the area of helicopter rotor flow analysis. This is, no doubt, due to the great complexity of the flow around a helicopterrotor in forward flight.

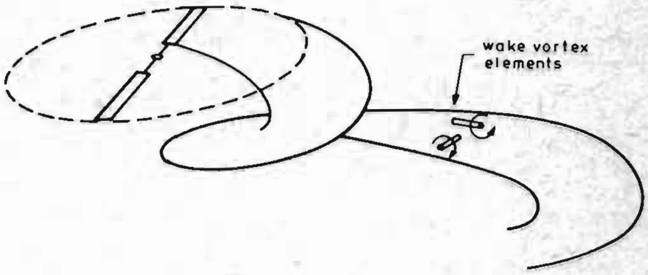

Fig. 1: Wake vorticity of a helicopterblade.

Figure 1 shows schematically the system of skewed helical vortex sheets trailed by a helicopter rotor. The vorticity in the wake consists of the socalled trailing vorticity, resulting from the spanwise variation of circulation along the blades, as well as the socalled shed vorticity, resulting from the time-variations of the blade circulation. In order to determine the induced velocity in points of the blade surface ("collocation points") it is usual to apply Biot and Savart's law, which requires a two-dimensional numerical integration over the skewed helical wake. To limit the amount of computational effort needed for a complete analysis of the time- and spanwise loading of a rotorblade, it is then essential to limit the number of collocation points to an absolute minimum. One tends to minimize especially the number of points along the bladechords, since it is well known that the spanwise variations of the loading along a rotorblade can be very rapid. One is thus forced by practical considerations to use, what might be called some type of "one point" lifting surface method, in other words Prandtl's lifting line theory or the 3/4-chord point method due to Weissinger.
Now every type of "one-point" method must necessarily be based upon an assumed type of chordwise load distribution. In both the existing lifting line methods the load distribution is assumed to be the one derived in two-dimensional, steady aerofoil theory. Application of one of the existing lifting line methods to the unsteady, sheared flow encountered by the sections of a rotorblade is therefore bound to stretch these classical methods beyond their limits of validity.

The purpose of the investigation described in this paper is, to develop a method for the analysis of the pressure distribution over the blades of a helicopterrotor which does not demand more than moderate numerical efforts, but which is nevertheless fully adapted to the special type of flow encountered by helicopterblades.

It will be shown that the description of an inviscid flow field by the method of the acceleration potential is almost ideally suited to this purpose. The acceleration potential, being in incompressible flow proportional to the pressure, does not show any discontinuities in the flowfield. This is in direct contrast with the more usual method of the velocity potential where the discontinuities (vortex sheets) play an essential role. When the blades are modelled into lifting lines (as far as their far field effect is concerned), the absence of discontinuities in the field permits the complete pressure field of the rotor to be expressed analytically as the field due to a set of pressure-dipole lines. The evaluation of the velocity in some point of the flow at a certain instant of time is equivalent to the computation of the velocity acquired by a particle of air travelling through the known pressure field and passing the considered point at the required time. The computation of the induced velocities along a rotorblade thus requires only a one-dimensional integration of the equations of motion with respect to time instead of the two-dimensional spatial integration over the helical vortex sheets needed in the velocity method. This results in considerable savings in computing time.
The absence of sheets of discontinuity also facilitates the derivation of the blade's near field, by means of a "matched asymptotic expansion" procedure. Such a systematic derivation instead of the more usual intuitive one has the advantage of leading almost automatically to the form which lifting line theory should take under the special circumstances met in a rotorblade analysis. The asymptotic procedure may furthermore be used to derive a higher order lifting line theory. The latter may be of special importance in relation to helicopter analysis: although in general the blades have large geometrical aspect ratio's, the flow is aerodynamically more comparable to a relatively small aspect ratio case, because of the rapid spanwise variations of the loading.
The paper concentrates on the fundamentals of the method of analysis mentioned above. For this reason great emphasis is given to the derivation of classical lifting line theory in the simple case of an uncambered wing in uniform motion, and to the modifications of the theory needed to account for sheared flow, unsteady conditions and higher order approximations respectively. The consequent application of the derived methods to the actual rotorblade will be self-evident in principle, and is not treated in great detail. The reader is furthermore referred to the original work (ref. 1) for more details, proofs and further extensions of the theory.

3. Brief review of the theory of the acceleration potential

The acceleration potential was first introduced in 1936 by Prandtl for the analysis of lifting surfaces in incompressible flow. The quantity $-\frac{p}{\rho}(x,y,z)$ was called the "acceleration potential" of the flow, since according to Euler's equation

$$\frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + (\underline{V}.\underline{\nabla})\underline{V} = -\operatorname{grad}(\frac{p}{\rho}) \qquad (1)$$

the gradient of $-p/\rho$ equals the acceleration of the fluid particles. Writing $\underline{V} = \underline{U} + \underline{V}'$ and $p = p_\infty + p'$ where $\underline{U}$ is the undisturbed velocity (taken to be independent of the space- and time coordinates) and $\underline{V}'$ is the perturbation velocity, linearization of Euler's equation leads to

$$\frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}'}{\partial t} + (\underline{U}.\underline{\nabla})\underline{V}' = -\frac{1}{\rho}\operatorname{grad} p' \qquad (2)$$

which yields, on taking the divergence of all the terms of (2) and applying the continuity equation $\operatorname{div}\underline{V}' = 0$, the Laplace equation for p':

$$\operatorname{div}\operatorname{grad} p' = \frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} + \frac{\partial^2 p'}{\partial z^2} = 0 \qquad (3)$$

In the following the primes will be omitted for convenience, so that p and $\underline{V}$ will both denote perturbation quantities.

Eq. (2) expresses the fact that in the linearized theory considered here, the velocity in a point of the field is found by integrating the acceleration of a particle of air coming from far upstream, whilst during this integration the particle's trajectory may be approximated by its straight, unperturbed trajectory. Boundary conditions must accordingly be applied to flat surfaces, parallel to the undisturbed flow.
In incompressible flow fields the pressure perturbation p cannot display any discontinuities except on the solid boundaries of the field. This is the main advantage of the pressure formulation: describing the field in terms of the pressure, no such things like free vortex sheets can enter into the mathematical formulation of the problem.

4. The pressure field of a flat plate aerofoil

As a preliminary the pressure field of a flat plate aerofoil in steady parallel flow will be considered (fig. 2).

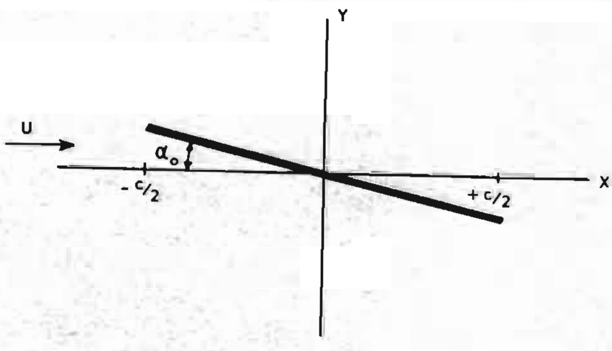

Fig. 2 : Flat plate aerofoil

In linearized theory particles of air moving along the surface of the aerofoil do not experience an acceleration in Y-direction so that according to

Euler's equation $\partial p/\partial y$ should be zero on the aerofoil, except at the very leading-edge where the streamline kink implies an infinite acceleration of the particles and hence also a pressure singularity. The boundary value problem for the pressure field thus becomes:

$$\left.\begin{aligned} & \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0, \\ & p \to 0 \text{ for } x^2 + y^2 \to \infty \\ & \partial p/\partial y = 0 \text{ on the aerofoil, i.e. on the stretch of the X-axis between } x = -c/2 \text{ and } x = +c/2 \\ & p \to -\infty \text{ at the leading edge, such that } v/U = -\alpha_o \text{ on the aerofoil} \end{aligned}\right\} \quad (4)$$

Kutta's condition of smooth flow at the trailing edge is implied by the above boundary value problem, since a singularity of $\partial p/\partial y$ has been required only at the leading edge. For the solution of this and subsequent problems, it is very convenient to use an elliptical coordinate system (fig. 3),

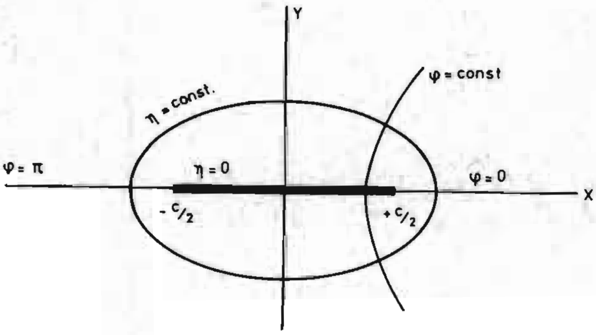

Fig 3: Elliptical coordinate system

conforming to the transformation formulae

$$x = c/2 \cosh\eta \quad \cos\varphi \qquad (5)$$

$$y = c/2 \sinh\eta \quad \sin\varphi \qquad (6)$$

Surfaces of constant η are ellipses, whereas the aerofoil is identical to the coordinate surface $\eta = 0$. Surfaces of constant φ are hyperbolae, orthogonal with the ellipses. The problem as stated in (4) reads, transformed into elliptic coordinates:

$$\left.\begin{aligned} & \frac{\partial^2 p}{\partial \eta^2} + \frac{\partial^2 p}{\partial \varphi^2} = 0 \\ & p \to 0 \text{ for } \eta \to \infty \\ & \partial p/\partial \eta = 0 \text{ for } \eta = 0 \\ & p \to -\infty \quad \text{for } \eta = 0 \text{ and } \varphi = \pi \end{aligned}\right\} \quad (7)$$

The solution, as may be checked easily, is given by the following pressure field:

$$\frac{p}{\frac{1}{2}\rho U^2} = -\frac{c_l}{\pi} \frac{\sin\varphi}{\cosh\eta + \cos\varphi} \qquad (8)$$

where c_l is the liftcoefficient of the aerofoil, equal to $2\pi\alpha_o$ according to the well-known results of velocity potential theory. What is very important for subsequent developments, is a consideration of the behaviour of the pressure field at large distances from the aerofoil. For large values of η the elliptical coordinate system degenerates asymptotically into a circular coordinate system (fig.4),

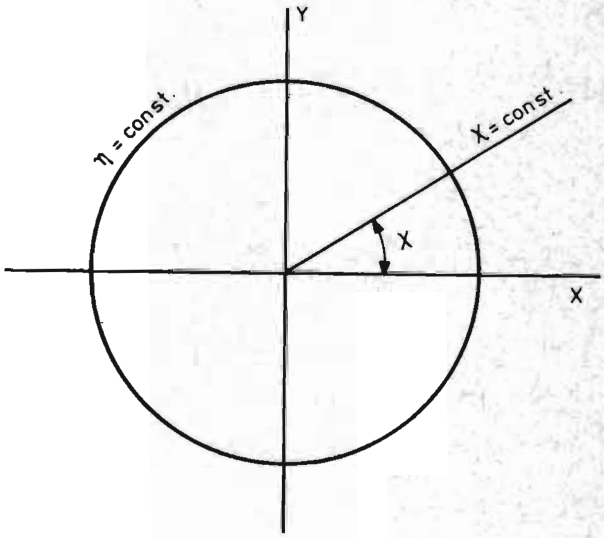

Fig. 4 Circular cylinder system.

with

$$\left.\begin{aligned} & c/2 \cosh\eta \approx c/2 \sinh\eta \approx \frac{c}{4} e^{\eta} \to r \\ & \varphi \to \chi \end{aligned}\right\} \text{for } \eta \to \infty \quad (9)$$

The pressure field of the flat plate aerofoil can then be shown to degenerate at large distances to

$$\frac{p}{\frac{1}{2}\rho U^2} \to -c_l c \frac{\sin\chi}{2\pi r} + c_m c^2 \frac{\sin 2\chi}{2\pi r^2} + \ldots \qquad (10)$$

$$\ldots \text{ for large } r$$

where the first term of the far field expansion is the field of a discrete dipole representing the lift of the aerofoil, and the second term is a quadrupole representing the pitching moment about the mid-chord point. The next term would <u>not</u> be, as perhaps expected, a discrete octupole. Equating $\cosh\eta$ to $\sinh\eta$ as is done in (9) implies neglecting the factor $e^{-\eta} \approx \frac{c/4}{r}$ with respect to the factor $e^{\eta} \approx \frac{r}{c/4}$.

Relative errors of the order $(c/r)^2$ are thus incurred in approximating the elliptical system by a circular one, and the octupole field would therefore have the same order of magnitude as this error. If one would expand the pressure field (8) further than two terms, one would no longer find discrete poles for the higher order terms.

5. Classical lifting line theory, as considered from the point of view of the pressure method

The linearized boundary value problem for an uncambered wing with span b and chord c (fig. 5)

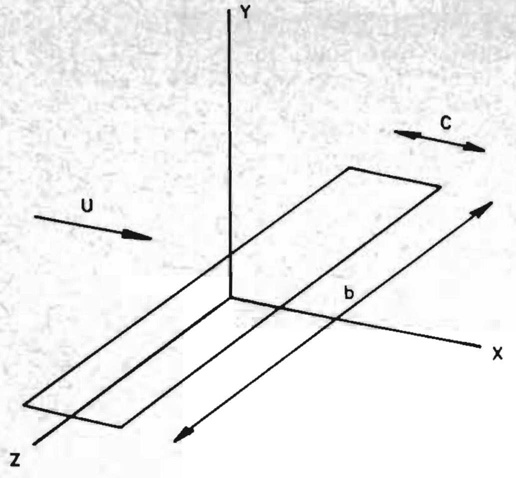

Fig.5: Notations straight, rectangular wing.

may be formulated as follows:

$$\left.\begin{aligned} &\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \\ &p \to 0 \text{ for } x^2 + y^2 + z^2 \to \infty \\ &\partial p/\partial y = 0 \text{ on the wingsurface} \\ &p \to -\infty \text{ along the leading edge, such that} \\ &\quad v/U(z) = -\alpha_o(z) \text{ on the wing} \end{aligned}\right\} \quad (11)$$

Instead of trying to find an exact solution for this problem, we will try to determine an approximation for the pressure field arround the wing, to a predetermined order of accuracy.
On physical grounds the assumption seems justified that in the immediate vicinity of the lifting surface, staying away from the tip regions, the characteristic length scale for spanwise pressure variations is the wingspan b, whereas the characteristic length of chordwise pressure variations is the chordlength c. Writing the Laplace-equation in terms of the characteristic coordinates $\frac{x}{c/2}$, $\frac{y}{c/2}$ and $\frac{z}{b/2}$ leads to

$$\frac{\partial^2 p}{\partial\left(\frac{x}{c/2}\right)^2} + \frac{\partial^2 p}{\partial\left(\frac{y}{c/2}\right)^2} = -\frac{1}{A^2}\frac{\partial^2 p}{\partial\left(\frac{z}{b/2}\right)^2} \qquad (12)$$

in which equation the three partial derivatives of p may be assumed to be of an equal order of magnitude, in accordance with the physical assumption introduced above. Eq. (12) shows that for wings of very large aspect ratio ($A \to \infty$) the pressure field close to the wingsurface (the socalled "near field") satisfies a two-dimensional Laplace-equation. In order to increase the accuracy of the analysis, one may attempt to describe also the rate at which the pressure field becomes two-dimensional for $A \to \infty$:

$$p(x,y,z)=p_o(x,y,z)+\frac{1}{A}\,p_1(x,y,z)+\frac{\ln A}{A^2}p_2(x,y,z)+ + \frac{1}{A^2}\,p_3(x,y,z)+ \ldots \text{ for } A\to\infty \qquad (13)$$

where p_o is the two-dimensional pressure field. The particular form chosen for the asymptotic behaviour will be substantiated later. Substituting the series (13) into the Laplace-equation (12) and multiplying successively by A, $A^2\ \ln^{-1}A$, A^2, etc., one finds on taking each time the limit $A\to\infty$ the following equations to be satisfied by $p_k(k=0,1,2,3)$:

$$\frac{\partial^2 p_k}{\partial\left(\frac{x}{c/2}\right)^2} + \frac{\partial^2 p_k}{\partial\left(\frac{y}{c/2}\right)^2} = 0 \quad (k=0,1,2) \qquad (14)$$

$$\frac{\partial^2 p_3}{\partial\left(\frac{x}{c/2}\right)^2} + \frac{\partial^2 p_3}{\partial\left(\frac{y}{c/2}\right)^2} = -\frac{\partial^2 p_0}{\partial\left(\frac{z}{b/2}\right)^2} \qquad (15)$$

In a theory accurate up to the order $O(A^{-1})$ the near pressure field $p_{near}(x,y,z)$ thus becomes equal to the pressure field of the flat plate aerofoil, except in the following respect. The assumption that the partial derivatives of p in the Laplace equation (12) are of equal order of magnitude can be valid only near the wingsurface, not too close to the wingtips. At larger distances from the wing the characteristic length scale for the pressure field may be assumed to be equal to b in all directions, so that the socalled "far pressure field" will certainly not satisfy a two-dimensional Laplace equation. For this reason we may not apply a condition at infinity to p_{near}. The field p_{near} may thus contain a part that does not vanish at large distances:

$$\frac{p_{near}}{\frac{1}{2}\rho U^2} = -\frac{c_1(z)}{\pi}\,\frac{\sin\varphi}{\cosh\eta + \cos\varphi} + \sum_{n=1}^{\infty} a_n(z)\cosh(n\eta)\sin(n\varphi)$$

The far pressure field $p_{far}(x,y,z)$ satisfies, as already explained, the full three-dimensional Laplace equation. When relative errors of order $O(A^{-2})$ are allowed, p_{far} may still be simplified however, namely to the field of a lifting line, i.e. a line along which pressure dipoles, quadrupoles etc. are distributed. This may be seen from the preceding paragraph where it was shown how the singularities distributed along the chord of an aerofoil at large distances seem to shrink into discrete singularities. Now this asymptotic behaviour of the pressure field was valid only when allowing relative errors of the order $(\frac{r}{c})^{-2}$. In the three-dimensional case we mean by "far field" the part of space at distances r from the wing that are of the same order as the span b. In points of this far field we "see" the wing at its correct span, while the wing chords seem to have shrunk into points carrying discrete singularities. The lifting line model of the far field is therefore equivalent to expressing our willingness to accept relative errors of the order $(b/c)^{-2} = A^{-2}$.

By allowing errors of order A^{-2}, we have obtained now two approximations for the pressure field around the wing, one that is valid near the wing, and one that is valid far from the wing. Both these approximations are still undetermined. In order to complete the problem, one must apply a "matching condition". This condition may be derived by requiring that it shall be possible to build up from the near- and far field a composite field, which is uniformly valid throughout the flow (fig. 6).

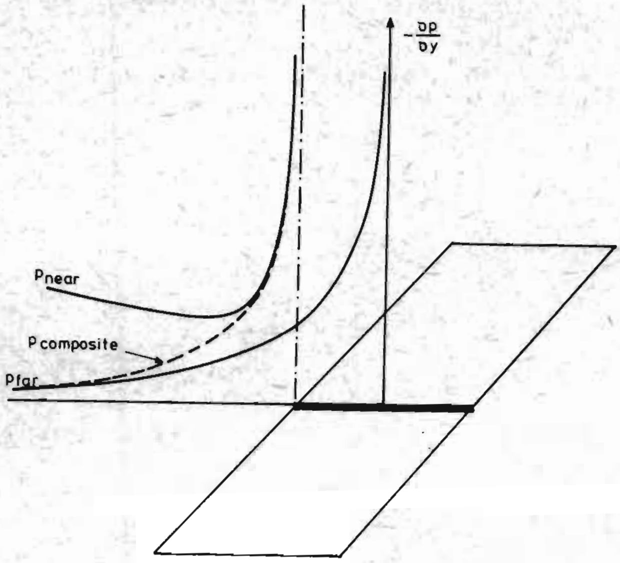

Fig. 6: Composite pressure field

Such a composite field may be formed by summing the near- and far pressure field, and subtracting a so-called "common field" which at large distances from the wing is identical (to the required order of accuracy) to the near field at large distances, and which is close to the wing surface identical to the far field.
Without going into the proof of existence of such a "common field", it is clear that if such a field exists, then we must require that the functions $\lim_{r\to Order\ (c)} p_{far}$ and $\lim_{r\to Order\ (b)} p_{near}$ become equal to the same function, so that also:

$$\lim_{r\to Order\ (b)} p_{near} = \lim_{r\to Order\ (c)} p_{far} \quad (17)$$

Now the near field behaves at distances of order b/2, neglecting terms of the order $O(A^{-2})$, like:

$$\frac{p_{near}}{\frac{1}{2}\rho U^2} \to -c_1(z)\ c\ \frac{\sin\chi}{2\pi r} + a_1(z)\sin\chi\,\frac{r}{c/2} + \ldots \quad (18)$$

Apparently, this field can be matched only to the field of a dipole-line $p_{dip}(r,\chi,z)$, having a distribution of dipole strength equal to $-c_1(z).c$. It can be shown that the field of such a dipole-line behaves at small distances from the wing like

$$\frac{p_{dip}}{\frac{1}{2}\rho U^2} \to -c_1(z)c\,\frac{\sin\chi}{2\pi r} + O(A^{-2}\ln A) \quad (19)$$

for $r \to$ Order $(c/2)$

which shows that in a theory which allows errors of $O(A^{-2})$, $a_1(z)$ must be zero. Both the near- and far field are now completely determined, and the composite field becomes:

$$\frac{p_{comp}}{\frac{1}{2}\rho U^2} = -\frac{c_1(z)}{\pi}\,\frac{\sin\varphi}{\cosh\eta+\cos\varphi} + \frac{p_{dip}}{\frac{1}{2}\rho U^2}(r,\chi,z) + c_1(z)\ c\ \frac{\sin\chi}{2\pi r} \quad (20)$$

We have thus obtained an expression for the pressure field around an uncambered wing. Wat is more, the expression can be put into closed form by expressing the field of a dipole-line like

$$\frac{p_{dip}}{\frac{1}{2}\rho U^2}(r,\chi,z) = \frac{\sin\chi}{2\pi}\sum_{n=1}^{\infty} A_n P_n^1(\cos\theta)Q_n^1(\cosh\psi) \quad (21)$$

where $P_n^1(x)$ and $Q_n^1(x)$ are associated Legendre-functions of the first- and second kind respectively. The "prolate spheroidal" (fig. 7) coordinates

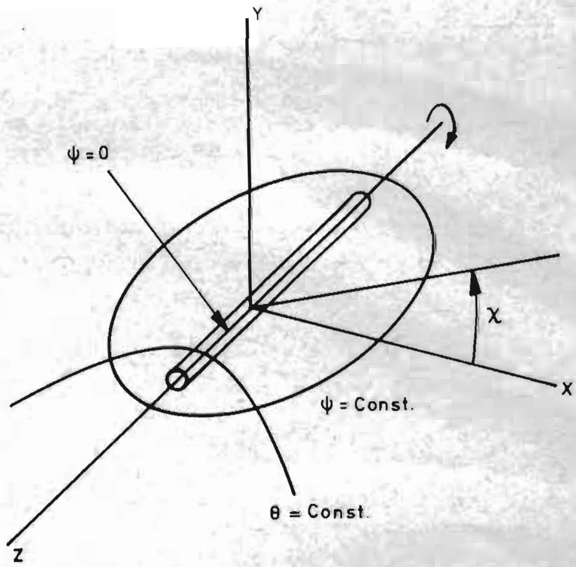

Fig. 7: Prolate spheroidal coordinates

θ and ψ are defined by

$$r = b/2\ \sinh\psi\ \sin\theta$$
$$\chi = \chi \quad (22)$$
$$z = b/2\ \cosh\psi\ \cos\theta$$

Surfaces of constant ψ are in this system ellipsoids, with $\psi = 0$ representing the lifting line. The surfaces of constant θ are hyperboloids orthogonal to the ψ = constant surfaces. In order to represent by (21) the field of a dipole-line which behaves as indicated by eq. (19), one must choose the coefficients A_n as:

$$A_n = \frac{2n+1}{n(n+1)}\,\frac{1}{A}\int_{-1}^{+1}\frac{c_1(\frac{z}{b/2})}{\{1-(\frac{z}{b/2})^2\}^{\frac{1}{2}}}\,P_n^{\,1}(\frac{z}{b/2})\,d(\frac{z}{b/2}) \qquad (23)$$

Legendre functions are, thanks to certain recurrence relations between them, easily and efficiently evaluated numerically. Expression (20) can thus be put into an efficient, closed form expression for the whole pressure field around the wing.
The resulting expression (20) is equivalent to classical lifting line theory, as will be shown now. If the wing is placed in a uniform stream perpendicular to the wingspan, with undisturbed velocity U in X^+-direction (fig. 5), then according to the linearized Euler equations (2):

$$U\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (24)$$

so that the velocity perturbation in Y^+-direction occurring along the mid-chord line of the wing follows from the equation

$$\frac{v}{U}(0,0,z) = -\frac{1}{\rho U^2}\int_{-\infty}^{o}\frac{\partial}{\partial y}(p_{comp})\,dx \qquad (25)$$

The first term (the two-dimensional term) in the right hand side of the composite pressure field (20) gives rise to v/U of a two-dimensional aerofoil, i.e. to $-\frac{c_1}{2\pi}$.It may be shown that the second and third term of (20) lead to $-v_i/U$, where v_i is, what is called in Prandtl's theory the induced velocity. Equating in accordance with the boundary condition (12) $v/U(0,0,z)$ to $-\alpha_o(z)$ the velocity integration (25) yields after some rearrangement

$$c_1(z) = 2\pi\,\{\,\alpha_o(z) - v_i/U(z)\,\} \qquad (26)$$

which is Prandtl's classical integral equation, stating that a wingsection behaves like a two-dimensional aerofoil placed at an effective angle of attack $\alpha_o - v_i/U$. The errors in classical lifting line theory are thus shown to be of the order $O(A^{-2})$.

All this may seem a rather large detour to find back a long known method for analyzing the lift distribution along straight wings. However, the present alternative formulation of lifting line theory has several advantages:

1) The pressure formulation of lifting line theory requires a one-dimensional numerical integration for the evaluation of v_i/U, a feature that is always retained no matter how complicated the flow becomes. Although this is a disadvantage in the simple case of a straight wing in parallel flow (where v_i/U can be found completely analytically using the velocity method) it becomes very convenient in cases where the vortex sheets are of such a complicated shape that a two-dimensional numerical integration over the vortex sheets would be needed.
2) The systematic rather than intuitive derivation of lifting line theory enables us to derive just as easily the form of lifting line theory applicable to more complicated situations.
3) The systematic derivation furthermore points out almost automatically how a higher-order theory may be developed.

6. Lifting line theory of the swept wing

We assume again a parallel flow in X^+-direction, where the rectangular wing is now rotated with respect to the undisturbed flow over the sweep angle Λ (fig. 8).

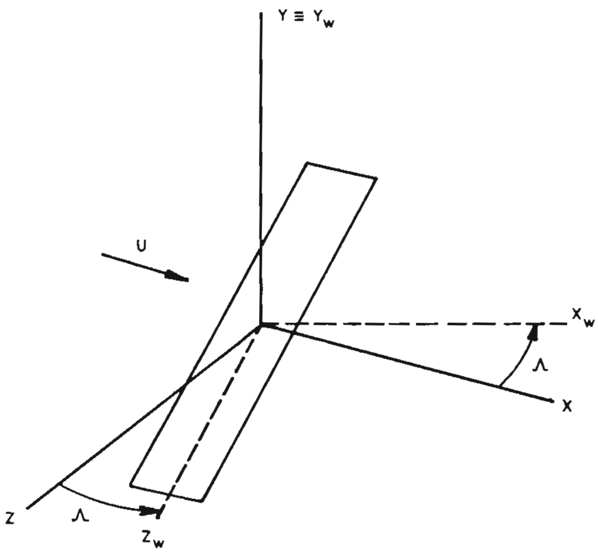

Fig. 8: Notations swept, rectangular wing.

In the wing-fixed coordinates (x_w, y_w, z_w) the surface of the uncambered, though twisted wing is given by

$$y = y_w = -\alpha_o\,(z_w)x_w \qquad |x_w| \leqslant c/2 \qquad (27)$$

giving also the Y-coordinate of a particle of air moving along the wing surface. The Y-component of the particle's velocity is then:

$$v = \frac{Dy}{Dt} = \frac{\partial y}{\partial x_w}\dot{x}_w + \frac{\partial y}{\partial z_w}\dot{z}_w =$$

$$= -\alpha_o(z_w)\;U\cos\Lambda - \frac{d\alpha_o}{dz_w}\,x_w\;U\sin\Lambda \qquad (28)$$

and the Y-component of its acceleration is derived likewise:

$$\frac{Dv}{Dt} = -\frac{d\alpha_o}{dz_w}U^2\sin 2\Lambda - \frac{d^2\alpha_o}{dz_w^{\,2}}\,x_w\;U^2\sin^2\Lambda \qquad (29)$$

Equating Dv/Dt to $-\frac{1}{\rho}\frac{\partial p}{\partial y}$, the boundary value problem for the swept wing becomes

$$\left.\begin{aligned}
&\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \\
&p \to 0 \text{ for } x^2+y^2+z^2 \to \infty \\
&\frac{\partial p/\partial y}{\rho U^2} = \frac{d\alpha_o}{dz_w} \sin 2\Lambda + \frac{d^2\alpha_o}{dz_w^2} x_w \sin^2\Lambda \text{ on the wing surface} \\
&p \to -\infty \text{ along the leading edge, such that along the mid-chord line } v/U(0,0,z_w) = -\alpha_o(z_w)\cos\Lambda
\end{aligned}\right\} (30)$$

This boundary value problem is very similar to that of the unswept wing, except that $\partial p/\partial y$ is non-zero, despite the absence of camber of the wing surface. The non-zero value of $\partial p/\partial y$ on the wing gives rise to an additional, non-singular pressure field, given by

$$\frac{p_2}{\frac{1}{2}\rho U^2} = -\frac{2}{A}\frac{d\alpha_o}{d(\frac{z_w}{b/2})} \sin 2\Lambda\, e^{-\eta} \sin\varphi + O(A^{-2}) \qquad (31)$$

The near field associated with the second spanwise derivative in (30), and all the far field terms are given by terms of order A^{-2}, so that these may be neglected to the order of accuracy achieved by lifting line theory. The complete solution of the boundary value problem is thus given by the sum of p_2 and the singular composite pressure field derived in the preceding section:

$$\frac{p}{\frac{1}{2}\rho U^2} = -\frac{c_{1_1}(z_w)}{\pi}\frac{\sin\varphi}{\cosh\eta + \cos\varphi} + \frac{\sin\chi}{2\pi}\sum_{n=1}^{\infty} A_n P_n^1(\cos\theta)\, Q_n^1(\cosh\psi) + c_{1_1}(z_w)c\frac{\sin\chi}{2\pi r} - \frac{2}{A}\frac{d\alpha_o}{d(\frac{z_w}{b/2})}\sin 2\Lambda\, e^{-\eta}\sin\varphi \qquad (32)$$

where c_1 still denotes the sectionwise liftcoefficient $l/(\frac{1}{2}\rho U^2 c)$ and the indexed symbol c_{1_1} indicates that part of the total c_1 of a section which is associated with the singular part of the pressure field.

In order to derive an integral equation for the unknown function $c_{1_1}(z_w)$, one must compute the mid-chord velocity perturbation $v/U(0,0,z_w)$ and equate this to $-\alpha_o(z_w)\cos\Lambda$. To find $v/U(0,0,z_{w_o})$ at the particular wingsection z_{w_o}, the pressure gradient is integrated as "experienced" by a particle of air, coming from infinity upstream and reaching the wingsection z_{w_o}. In linearized theory the particle's trajectory may be taken to coincide with its unperturbed trajectory, i.e. with a straight path, parallel to the X -axis. In wing-fixed coordinates:

$$\left.\begin{aligned}
x_w(t) &= U\,t\cos\Lambda \\
y_w(t) &= 0 \\
z_w(t) &= z_{w_o} + U\,t\sin\Lambda
\end{aligned}\right\} (33)$$

where the particle is assumed to reach the mid-chord line at $t = 0$. Using the transformation-formulae relating η,φ, r, χ,θ and ψ to x_w,y_w,z_w, the time-function $\partial p/\partial y(t)$ as experienced by the particle is found by differentiating the composite field (32) w.r. to y. The value of $v/U(0,0,z_{w_o})$ follows from the time-integration

$$v/U(0,0,z_{w_o}) = -\frac{1}{\rho U}\int_{-\infty}^{0}\frac{\partial p}{\partial y}(t)\,dt \qquad (34)$$

The integrand contains several singularities, namely at the times when the particle reaches the leading edge and the mid-chord line. Taking the necessary precautions (not considered any further in this paper) a numerical integration is easily performed.

Although the equations given above suffice to compute the pressure distribution over the wing, it is interesting to write out the expressions in a slightly different form. Firstly, one may split off from the composite pressure field (32) the purely two-dimensional field

$$-\frac{c_{1_1}(z_{w_o})}{\pi}\frac{\sin\varphi}{\cosh\eta + \cos\varphi},$$

which yields after integration the mid-chord downwash v/U of a two-dimensional aerofoil placed in a flow with unperturbed velocity $U\cos\Lambda$, i.e. $-\frac{c_{1_1}(z_{w_o})}{2\pi\cos\Lambda}$ (according to the definition of c_1 adopted here).

Writing the integral of the dipole-terms in (32) symbolically as $-v_i/U$ again, and rearranging, the following expression for the sectionwise lift distribution is obtained

$$c_{1_1}(z_{w_o}) = 2\pi\cos^2\Lambda\{\alpha_o(z_{w_o}) - \frac{v_i}{U\cos\Lambda}\} + \pi U\cos\Lambda\int_{-\infty}^{0}\left[\frac{\partial}{\partial y_w}\left[\frac{c_{1_1}\{z_w(t)\} - c_{1_1}(z_{w_o})}{\pi\{\cosh\eta(t) + \cos\varphi(t)\}}\right].\sin\varphi(t) + \frac{2}{A}\frac{d\alpha_o}{d(\frac{z_w}{b/2})}\{z_w(t)\}\sin 2\Lambda\, e^{-\eta(t)}\sin\varphi(t)\right]dt \qquad (35)$$

Often in helicopter analysis, one accounts for the sheared flow met by the blade sections, by considering only the velocity components perpendicular to the blade section before applying two-dimensional section data. This is the well-known $\cos\Lambda$ sweep correction. Eq. (35) now shows clearly that this procedure yields only one of the several corrections necessitated by the sheared flow. The integrals in (35) represent corrections of order $O(A^o)$ that have to be applied even in a low-accuracy method like classical lifting line theory. These additional corrections are automatically included in the lifting line method formulated in eqs. (32) to (34).

6. The helicopterrotor in forward flight

The situation is shown schematically in fig. 9.

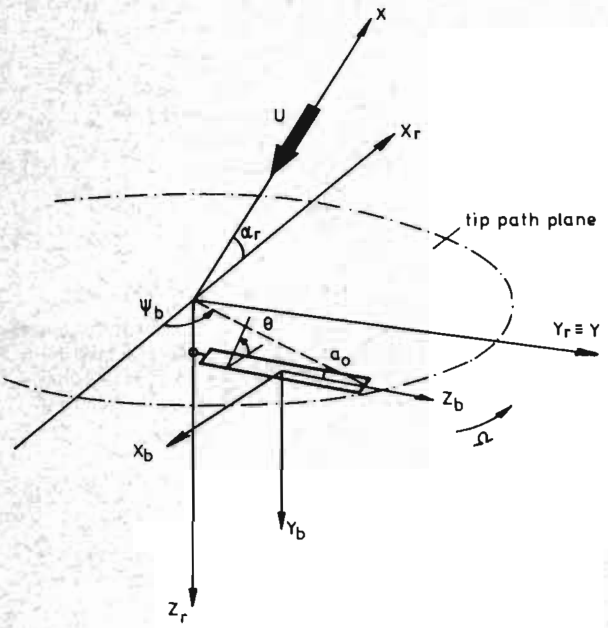

Fig. 9: Notations helicopter rotorblade

The uncambered blades are assumed to have a rectangular planform and an amount of linear twist ε. Whilst rotating around the azimuth (denoted by the angle ψ_b) with angular velocity Ω, the blade sections execute a harmonic pitching motion with respect to the tip path plane, given by

$$\theta_{section}=\theta_o-\varepsilon\frac{r_b}{R}+b_1\cos\psi_b-a_1\sin\psi_b \qquad (36)$$

where θ_o = collective pitch angle of blade root with respect to the tip path plane,

r_b = radial distance of blade element from rotorhub,

a_1 and b_1 correspond to the usual notations for the unit flapping angles with respect to the control plane of the rotor.

The blades include furthermore a coning angle a_o with the tip path plane. The free-stream velocity U is directed at an angle of attack α_r to the tip path plane.

Despite the complicated geometry indicated above, the boundary value problem for the instantaneous pressure field around a rotorblade as expressed in the blade-fixed coordinates (x_b,y_b,z_b) is not much different from that of a wing in uniform motion:

$$\left.\begin{array}{l}
\dfrac{\partial^2 p}{\partial x_b^2}+\dfrac{\partial^2 p}{\partial y_b^2}+\dfrac{\partial^2 p}{\partial z_b^2}=0\\
\text{on the blade surface:}\\
-\dfrac{1}{\rho\Omega^2 R}\dfrac{\partial p}{\partial y_b}=F_1(\psi_b,z_b)+\dfrac{x_b}{R}F_2(\psi_b,z_b)\\
p\rightarrow 0 \quad \text{for } x_b^2+y_b^2+z_b^2\rightarrow\infty\\
p\rightarrow\infty \text{ along the leading edge, such that along the mid-chord line}\\
\dfrac{w}{\Omega R}=F_3(\psi_b,z_b)
\end{array}\right\} \qquad (37)$$

The precise form of the functions F_1, F_2 and F_3 will be found in the original work (ref. 1).
The essential differences with the straight wing in parallel flow are:

a) p is the instantaneous pressure perturbation field, with the boundary conditions depending upon the instantaneous azimuth angle of the blades,

b) $\partial p/\partial y_b$ is non-zero, even though the blades are assumed to be uncambered.

The complete solution of the boundary value problem is given by a pressure field which is instantaneously very similar to the one found in the case of the swept wing:

$$\frac{p}{\rho\Omega^2R^2}=\frac{c_{t_1}(\psi_b,z_b)}{\pi}\,\frac{\sin\varphi}{\cosh\eta+\cos\varphi}+$$

$$-\frac{\sin\chi}{2\pi}\sum_{n=1}^{\infty}A_n(\psi_b)P_n^1(\cos\theta)\,Q_n^1(\cosh\psi)$$

$$-c_{t_1}(\psi_b,z_b)\,c\,\frac{\sin\chi}{2\pi r}+\frac{F_1(\psi_b,z_b)}{2\,A}\sin\varphi\, e^{-\eta} \qquad (38)$$

where c_t is defined as $c_t=\frac{\ell}{\rho\Omega^2R^2c}$ (note that ℓ is taken positive in negative Y_b-direction), and c_{t_1} is that part of the total c_t of a section which is associated with the singular pressure field. The coefficients $A_n(\psi_b)$ are given by

$$A_n(\psi_b)=\frac{2n+1}{n(n+1)}\,\frac{1}{A}\int_{-1}^{+1}\frac{c_{t_1}(\psi_b,z_b)}{\{1-(\frac{z_b}{R/2})^2\}^{\frac{1}{2}}}\,\cdot$$

$$\cdot\,P_n^1\left(\frac{z_b}{R/2}\right)d\left(\frac{z_b}{R/2}\right) \qquad (39)$$

Summing the pressure field (38) over all the blades of the rotor yields an easily computable, closed form expression for the pressure field of a helicopter rotor.
In order to derive an integral equation for the unknown function $c_{t_1}(\psi_b,z_b)$, one must compute the

mid-chord velocity perturbation $\frac{w}{\Omega R}(\psi_b, z_b)$ and equate it to $F_3(\psi_b, z_b)$ in accordance with the boundary value problem (37). To find $\frac{w}{\Omega R}$ at the particular blade section z_{b_o} when the blade is in the azimuth position ψ_{b_o}, one must integrate the pressure gradient "experienced" by a particle of air, coming from infinity upstream and reaching the blade section at time $t_o = \frac{\psi_{b_o}}{\Omega}$. In linearized theory the coordinates of the considered particle are found as a function of time from its unperturbed trajectory:

$$\left.\begin{aligned} x(t) &= x_o - U(t-t_o) \\ y(t) &= y_o \\ z(t) &= z_o \end{aligned}\right\} \quad (40)$$

where x_o, y_o, z_o are the coordinates of the mid-chord point of the considered section at t_o, expressed in the "flow-fixed" coordinate system (x,y,z) (fig. 9). Now a set of transformation formulae may be derived to relate the flow coordinates (x,y,z) to the blade coordinates (x_b, y_b, z_b) so that the functions $x_b(t)$, $y_b(t)$, $z_b(t)$ pertaining to the particle are known. This means that also $p/(\rho\Omega^2R^2)$ (t) and $\partial p/\partial y(t)$ as experienced by the particle are known as a function of time, and the time-integration

$$\frac{w}{\Omega R} = -\frac{1}{\rho\Omega R}\int_{-\infty}^{t_o}\frac{\partial p}{\partial y_b}(t)\,dt \quad (41)$$

then yields the sought velocity perturbation.
Note that $\frac{w}{\Omega R}$ is found again by performing a one-dimensional numerical integration with respect to time, of an integrand which is evaluated readily. This contrasts to the two-dimensional spatial integration over the skewed helical vortex sheets needed in the usual velocity method.
Another interesting point shows up by reconsidering eq. (38). In analogy with the earlier treated case of the swept wing, one might again split off from the first term in the right hand side a pressure field:

$$\frac{c_{t_1}\{\psi_b(t), z_{b_o}\}}{\pi}\,\frac{\sin\varphi}{\cosh\eta+\cos\varphi} \quad (42)$$

The pressure field (42) represents the field associated with a two-dimensional aerofoil whose lift is a periodic function of time. The time-varying mid-chord downwash $\frac{w_1}{\Omega R}(t)$ associated with the field (42) depends on the time-function $c_{t_1}(t)$ via a two-dimensional functional relationship:

$$\frac{w_1}{\Omega R}(t) = f_{\text{two-dim}}\left[c_{t_1}(t)\right] \quad (43)$$

so that, analogous to the case of the swept wing, c_{t_1} may be expressed like:

$$c_{t_1}\{\psi_b(t_o), z_{b_o}\} = f^{-1}_{\text{two-dim}}\left[F_3\{\psi_b(t_o), z_{b_o}\} + -\frac{w_i}{\Omega R}\{\psi_b(t_o), z_{b_o}\}\right] + f^{-1}_{\text{two-dim}}\left[\Omega R\int_{-\infty}^{t_o}\frac{\partial}{\partial y_b}\,\frac{c_{t_1}\{\psi_b(t), z_b(t)\} - c_{t_1}\{\psi_b(t), z_{b_o}\}}{\pi\{\cosh\eta(t)+\cos\varphi(t)\}}\sin\varphi(t)\,dt + \Omega R\int_{-\infty}^{t_o}\frac{\partial}{\partial y_b}\,\frac{F_1\{\psi_b(t), z_b(t)\}}{2A}\,.\,\sin\varphi(t)\,e^{-\eta(t)}\,dt\right] \quad (44)$$

Often in helicopter analysis, one accounts for unsteady effects by calculating the periodically time-fluctuating effective angle of attack of the blade section, and equating the lift of the section to the lift as experienced by a two-dimensional aerofoil whose angle of attack varies in the same way as the effective angle of attack of the considered blade section. This procedure corresponds with the first term in the right hand side of (44). The present analysis indicates that this procedure leads to errors of the order $O(A^o)$, since it neglects the contribution of the other terms in the right hand side of (44). Clearly, errors of $O(A^o)$ cannot be allowed in a lifting line theory, which is accurate up to $O(A^{-1})$.
Now one could nevertheless use this simple - but incorrect - procedure hoping to get at least a qualitative picture of what happens near the point of stall of the blade sections. One would then determine the two-dimensional functional relationship (42) from two-dimensional experiments, in order to study phenomena like dynamic stall. Note, however, that in the case of the helicopter blade, the correct two-dimensional experiment would be a very awkward one: the pressure field (42) is the field of a two-dimensional flat-plate aerofoil which is itself fixed with respect to an inertial frame of reference, while the undisturbed stream velocity has a time varying direction with respect to the inertial frame (aerofoil moving through a gust-field). This kind of experiment is naturally hardly realizable. Much more practicable is the experiment where the two-dimensional aerofoil oscillates periodically with respect to an inertial frame of reference (the wind tunnel) whilst the undisturbed stream is fixed in direction with respect to the inertial frame. This procedure is indeed often followed in practice. One must then realize however, that by the oscillation of the aerofoil a second, non-singular pressure field is created as well. Especially when the interest is concentrated on studying boundary layer effects (dynamic stall), this second pressure field might have a quite desastrous effect upon the reliability of the two-dimensional experiment. One would never be certain about the influence of the second pressure field upon the boundary layer phenomena studied.

7. Higher-order lifting line theory

One of the features of the pressure method is, that a higher-order - more accurate - lifting line theory may be derived readily. This will be illustrated again for the case of the rectangular wing in steady parallel flow (fig. 5). As may be seen from the equations (14) and (15) of section 5, when terms of order A^{-2} are included, the near pressure field p_{near} must satisfy a two-dimensional Poisson equation:

$$\frac{\partial^2 p_{near}}{\partial(\frac{x}{c/2})^2} + \frac{\partial^2 p_{near}}{\partial(\frac{y}{c/2})^2} = -\frac{1}{A^2}\frac{\partial^2}{\partial(\frac{z}{b/2})^2}(p_{two\text{-}dim}) \quad (45)$$

The solution of (45) will consist of the sum of a particular solution of the Poisson-equation and solutions of the two-dimensional Laplace equation, and takes in the present case the form

$$\frac{p_{near}}{\frac{1}{2}\rho U^2} = -\frac{c_{1_1}(z)}{\pi}\frac{\sin\varphi}{\cosh\eta+\cos\varphi} + a_1(z)\cosh\eta\sin\varphi + \ldots +$$

$$+ \frac{1}{\pi A^2} c''_{1_1} (\tfrac{1}{2}\eta \sinh\eta \sin\varphi + \tfrac{1}{8}\sin 2\varphi) \quad (46)$$

where the indexed c_{1_1} indicates the c_1 associated with the singular part of the pressure field, which is only a part of the total c_1 of the wing section. The symbol c''_{1_1} denotes the second derivative of c_{1_1} with respect to the spanwise coordinate $\frac{z}{b/2}$. According to section 5, the far pressure field corresponds to a field of line singularities, if relative errors of order A^{-2} are allowed. Now from the preceding theory it appears that the leading term in the far field expression (i.e. the dipole singularity) is of the order A^{-1} with respect to the leading term in the composite pressure field. This shows, that for the purpose of building up a composite field accurate to order A^{-2}, the far field representation by a distribution of line singularities may still be used.

Now considering the behaviour of the near field at large distances (to order $O(A^{-2})$):

$$\frac{p_{near}}{\frac{1}{2}\rho U^2} \rightarrow -c_{1_1}(z)c\frac{\sin\chi}{2\pi r} + c_{1_1}(z)\frac{c^2}{4}\frac{\sin 2\chi}{2\pi r^2} +$$

$$+a_1(z)\frac{r}{c/2}\sin\chi + \ldots + \frac{1}{2\pi A^2}c''_{1_1}\ln(\frac{r}{c/4})\frac{r}{c/2}\sin\chi +$$

$$+\frac{1}{8\pi A^2}c''_{1_1}\sin 2\chi$$

$$\text{for } r \rightarrow \text{order } (b/2) \quad (47)$$

it is seen that the far field must now consist of a dipole-line as well as a quadrupole-line. These can be shown to behave close to the lifting line like:

$$\frac{p_{dip}}{\frac{1}{2}\rho U^2}(r,\chi,z) \rightarrow -c_{1_1}(z)c\frac{\sin\chi}{2\pi r} + \frac{1}{2\pi A^2}c''_{1_1}\cdot$$

$$\cdot \ln(\frac{r}{c/4})\frac{r}{c/2}\sin\chi + \frac{\sin\chi}{4\pi A}\frac{r}{c/2}G(\frac{z}{b/2}) +$$

$$+ O(A^{-4}\ln A) \qquad \text{for } r \rightarrow \text{order } (c/2) \quad (48)$$

$$\frac{p_{quad}}{\frac{1}{2}\rho U^2}(r,\chi,z) \rightarrow c_{1_1}(z)\frac{c^2}{4}\frac{\sin 2\chi}{2\pi r^2} + \frac{1}{8\pi A^2}c''_{1_1}\sin 2\chi +$$

$$+ O(A^{-4}\ln A) \qquad \text{for } r \rightarrow \text{order } (c/2) \quad (49)$$

showing that the only unknown function $a_1(z)$ is given to $O(A^{-2})$ by:

$$a_1(z) = \frac{G(\frac{z}{b/2})}{4\pi A} = 2\{\frac{p_{dip}}{\frac{1}{2}\rho U^2}(c/4,\pi/2,z) + \frac{2}{\pi}c_{1_1}(z)\} \quad (50)$$

These expressions suffice to build up the composite field. When actually using the composite field several simplifications can be introduced, such as the substitution of a single dipole-line shifted towards the quarter-chord position instead of the far field consisting of a dipole- as well as a quadrupole-line. Integrating the composite field finally yields the following results:

$$c_1(z) = c_{1_1}(z) - \pi\{\frac{p_{dip}}{\frac{1}{2}\rho U^2}(c/4,\pi/2,z) + \frac{2}{\pi}c_{1_1}(z)\} \quad (51)$$

$$c_m(z) = \frac{1}{4}c_{1_1}(z) + \frac{1}{64 A^2}c''_{1_1} \quad (52)$$

where $c_{1_1}(z)$ is determined by the integral equation

$$c_{1_1}(z) = 2\pi(\alpha_o - \frac{v_i}{U}) - \pi\{\frac{p_{dip}}{\frac{1}{2}\rho U^2}(c/4,\pi/2,z) +$$

$$+ \frac{2}{\pi}c_{1_1}(z)\} \quad (53)$$

One may show that the higher-order lifting line method developed here is equivalent to Weissinger's three-quarter chord method, in the case of wings with linear twist, placed in a steady parallel flow. For wings with a more general type of twist-distribution, Weissinger's method involves a slight, but probably insignificant error of order A^{-2}. An impression about the accuracy achieved by these higher-order lifting line methods may be obtained from fig. 10, taken from ref. 2. The figure shows the accuracy of the higher-order lifting line methods to be close to the accuracy of a full lifting surface method.

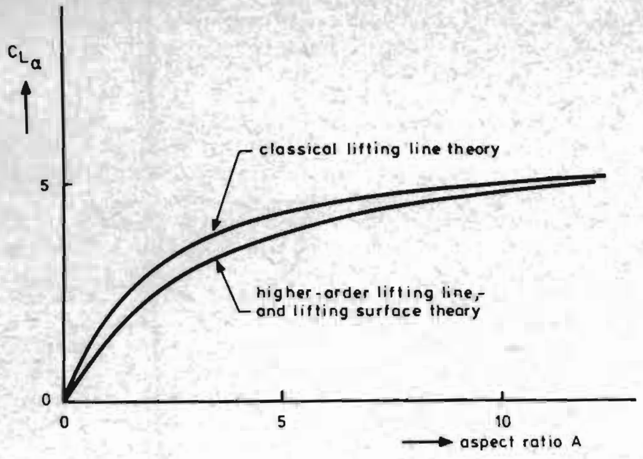

Fig.10: Computed C_{L_α} of rectangular wings, acc. to ref. 2.

The difficulties of applying, as is often done, Weissinger's method to helicopter analysis are:

1) the 3/4-chord method is not valid in unsteady cases,
2) the 3/4-chord method does not give information about the pressure distribution over the wing, so that pitching moments cannot be analyzed.

Both these difficulties have now been eliminated by deriving the higher-order method via en asymptotic expansion procedure. The composite pressure field may be used directly in unsteady cases, in the same way as shown for the classical lifting line theory. The pressure distribution over the helicopter blade is equally easily constructed.

A typical computed pressure distribution for a blade at the advancing side of the rotordisc is shown in fig. 11. The influence of the tipvortex of a preceding blade, passing underneath the considered blade at about 80% of the blade radius, can be easily recognized. Although such results as shown must be considered preliminary as yet, two important tentative conclusions can be drawn:

1) the physical assumptions underlying the lifting line approximations (section 5) can be seen to be justified up to blade stations very close to the blade tip. Considering the good accuracy of higher-order lifting line methods and the fact that it is possible to compute complete pressure distributions using the theory developed here, the use of a full lifting surface method would have hardly any advantage compared with the higher-order lifting line theory.
2) The higher-order terms, i.e. the terms of order A^{-2} rather strongly affect the pressure distribution in the tip region. Their effect is such, that for a given lift coefficient the leading edge pressure peak is increased, compared with the purely two-dimensional pressure distribution. This fact could have an important influence upon the high Mach-number characteristics of the blade sections near the tip.

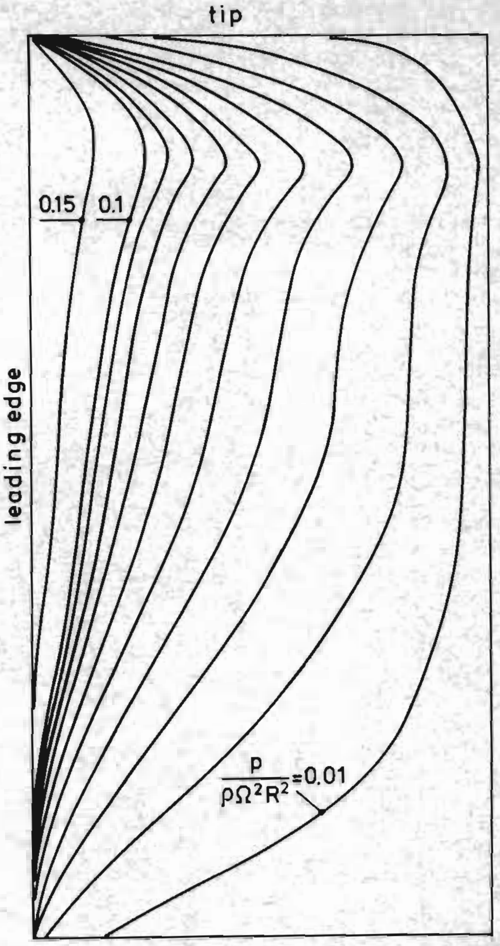

Fig. 11: Typical isobar contours on advancing blade (note: chord and span not drawn to same scale, real aspect ratio A=12.7)

8. Conclusions

1) The common practice in the classical lifting line analysis of helicopter blades to account for sheared flow by the socalled simple cosΛ sweep-correction is incorrect. It introduces errors of the order $O(A^0)$ into the lifting line analysis which itself may be accurate up to $O(A^{-1})$.

2) Consequently, it is incorrect to equate the unsteady lift of a blade section to the unsteady lift of a two-dimensional aerofoil which moves through a periodical gust field, where the gust distribution corresponds to the time-variations of the induced velocity. This also introduces errors of the order $O(A^0)$.

3) The flow around a blade section is not even qualitatively comparable with the flow around a periodically oscillating two-dimensional aerofoil placed in a windtunnel. Therefore, such wind tunnel experiments are of doubtful value. This is especially so when the interest is focused on unsteady boundary layer effects near the stall, such as "dynamic stall".

4) The 3/4-chord lifting line method can be proved to be exact up to the order $O(A^{-2})$ under certain conditions. These conditions are not satisfied however, when the 3/4-chord method is applied to helicopter blades. In that case errors are introduced of the order $O(A^{-1})$.

5) Using the theory of the acceleration potential in combination with a matched asymptotic expansion technique, lifting line methods can be developed for application to the helicopter blade which avoid all the above mentioned problems.

6) The methods mentioned under 5) are efficient in numerical computations, since the two-dimensional integrations over the skewed helical vortex sheets needed in the velocity method are reduced to one-dimensional integrations using the acceleration potential.

7) The matched asymptotic expansion treatment of lifting line theory yields the complete pressure distribution over the helicopter blades, which is an advantage over existing lifting line methods. Therefore, the good accuracy of higher-order lifting line methods would seem to indicate that little would be gained by developing full lifting surface methods for the helicopter blade.

8) Preliminary results indicate that the pressure distribution over the tip region of a blade deviates rather much from two-dimensional distributions. This observation could be of importance in relation to the development of blade sections for high critical Mach-numbers.

9. References

1) Th. van Holten: The computation of aerodynamic loads on helicopter blades in forward flight, using the method of the acceleration potential. Doctor's thesis Technological University Delft, to be published autumn 1974.

2) H. Schlichting, E. Truckenbrot: Aerodynamik des Flugzeuges, Springer-Verlag 1969.

DISCUSSION

W.P. Jones (Texas A & M University, College Station, Texas, U.S.A.): I congratulate the author on a very interesting paper and I would like to ask two questions:
1. How is the effect of wake distortion and mutual interference due to the effect of other blades taken into account?
2. Lifting line theory will not give the chordwise pressure distributions and pitching moment derivatives would not be given accurately. Does the author agree?

By way of further comment I might point out that M. Dat of Onera and Jones and Rao of Texas A & M University had worked on the same problem and taken compressibility effects into account.

Th. van Holten: 1. The mutual interference between the blades is automatically accounted for, by summing the pressure field of all the blades in order to arrive at the pressure field of the rotor as a whole. When a particle of air comes from infinity upstream, it will closely pass several of the blades before it finally reaches the point (in general: a collocation point on one of the blades) where the total velocity perturbation is calculated. When passing through the pressure field of any blade, the particle whose velocity perturbation is calculated experiences an acceleration. Therefore, all the blades contribute to the final velocity perturbation. In fact, it can be shown that the perturbation velocity thus obtained is the same as the perturbation velocity such as would be calculated using the more usual theoretical model involving vortex sheets.

The wake distortion is not taken into account in the linearized theory described here. It can be taken into account in a way which is indicated in the original work (ref. 1 of the ICAS-paper).
2. The usual lifting line theories indeed do not give information about chordwise pressure distributions. The lifting line theories described in the paper, derived by a matched asymptotic expansion method, on the contrary do give full information on the structure of the near pressure field, so that the pressure distribution along the chord is known (see for instance fig. 11 of the paper).